Starters and hints for 3.1 and 3.2

3.1 # 11-14

These questions are **exactly like** questions 7-10 (part c).

11) $f(x) = x^2 + 1$; $x_0 = 2$.

(a) Find the slope of the graph of *f* at a general *x*-value x_0.

The 'slope of the graph' is the 'slope of the tangent line' is the 'instantaneous rate of change' is defined to be 'the derivative of *f* at x_0'. So:

$$f'(x_0) = \lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0} = \lim_{x\to x_0}\frac{\left(x^2+1\right)-\left(x_0^2+1\right)}{x-x_0} = \lim_{x\to x_0}\frac{x^2-x_0^2}{x-x_0} = \lim_{x\to x_0}\frac{\left(x-x_0\right)\left(x+x_0\right)}{x-x_0} = \lim_{x\to x_0}\left(x+x_0\right) = 2x_0$$

Alternatively (we learned two definitions …)

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\left((x+h)^2+1\right)-\left(x^2+1\right)}{h} = \lim_{h\to 0}\frac{x^2+2hx+h^2+1-\left(x^2+1\right)}{h} =$$

$$\lim_{h\to 0}\frac{2hx+h^2}{h} = \lim_{h\to 0}\left(2x+h\right) = 2x$$

(b) Find the slope of the graph at the *x*-value specified by x_0.

When x_0=2, we have $f(x_0) = f(2) = 2(2) = 4$. It's much easier doing the general case (and finding the always-true rule) first!

---

15) is about interpreting the graph. We'll discuss that perhaps ….

3.2 #1

Use the graph of $y = f(x)$ in the accompanying figure to estimate the value of $f'(1), f'(3), f'(5), f'(6)$.

The value of these **derivatives** is the value of the slope of the tangent line at those points. So look at $x = 1$, and find $f(1) = 3$. Most of the your books, I'm sure, already have the lines drawn in – the book I'm using at home has them drawn in with a ruler! Remember – the tangent line is just going to "kiss" the curve at (1, 3) and in my book it looks like it also goes through (0, 1). That would make its slope = 2, and so our **estimate** is $f'(1) = 2$.

I'll also do the easiest one! Take a look at (3, 6) – that looks like a maximum value, and the tangent line looks like it would be horizontal. What is the slope of a horizontal line? **Zero** So again we estimate …
$f'(3) = 0$

And you try the rest ☺

4) Given that the tangent line to $y = f(x)$ at the point (-1, 3) pass through the point (0, 4), find $f'(-1)$.

Think about this ….. $f'(-1)$ is the derivative of *f* at -1. The derivative at -1 is the **same** number as the slope of the tangent line to *f* at -1. **But it tells us about the tangent line** at x = -1! In particular, we have two points on that tangent line, and with two points I can find the ……. Slope! SO ….

$m = \frac{\Delta y}{\Delta x} = \frac{4-3}{0-(-1)} = \frac{1}{1} = 1$. **AND** that is also the value of the derivative $f'(-1)$. Notice that it **is the value of derivative** – it is **not** an estimate in this case.

=> Hope this helps. We'll discuss tomorrow …..

Peace - Bob